

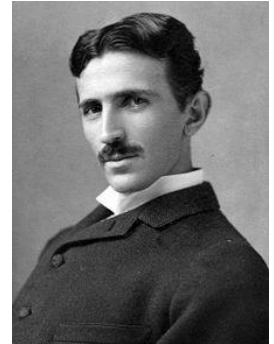
ELEKTROMOTORNI POGONI SA ASINHRONIM MOTOROM

Proučavamo samo pogone sa trofaznim motorom.

Najčešće korišćeni motor u elektromotornim pogonima.

Prednosti asinhronih motora:

- jednostavna konstrukcija;
- mala cena;
- visoka energetska efikasnost.



Nikola Tesla

1856-1943



Asinhroni (indukcioni) motor
Patent iz 1888 godine

METALNI PRSTEN

**LAMINIRANO
JEZGRO**

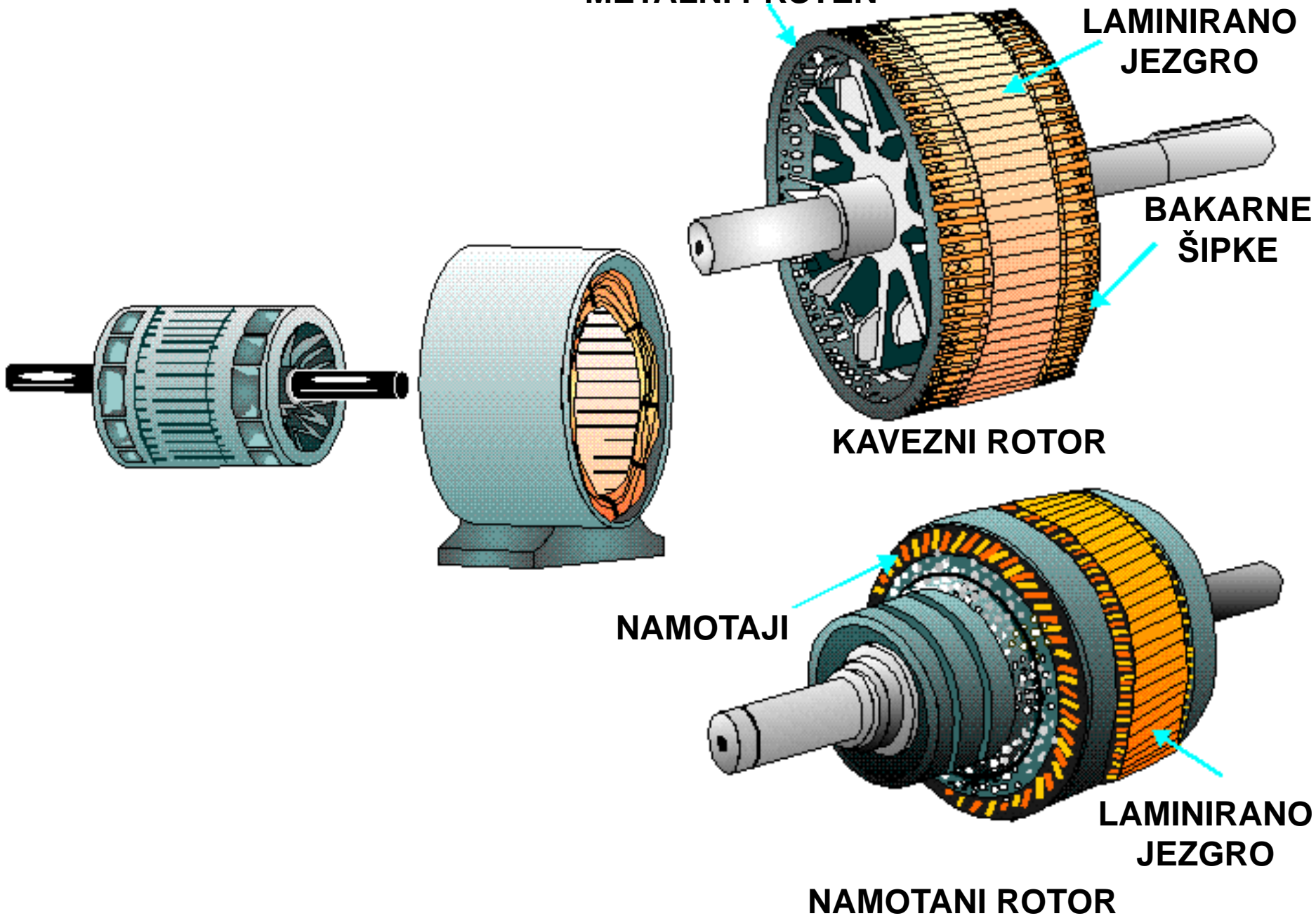
**BAKARNE
ŠIPKE**

KAVEZNI ROTOR

NAMOTAJI

**LAMINIRANO
JEZGRO**

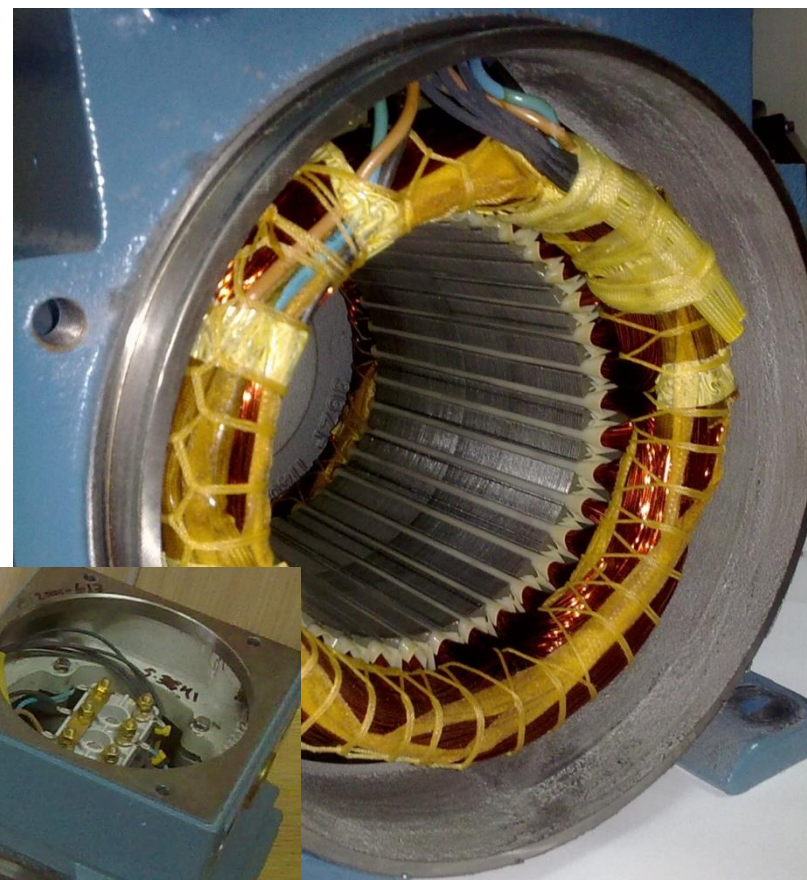
NAMOTANI ROTOR

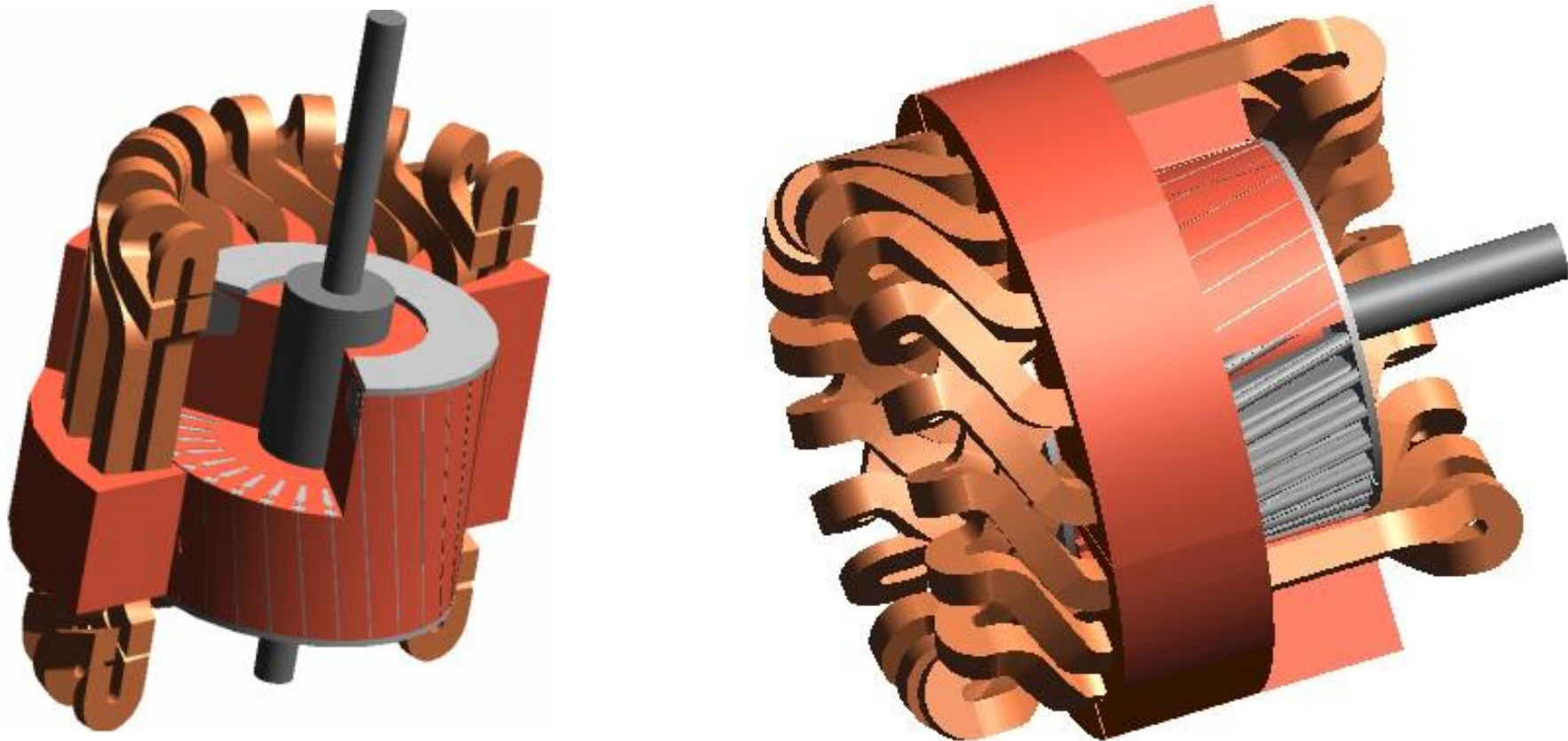


Motor sa kaveznim rotorom

Stator

Rotor





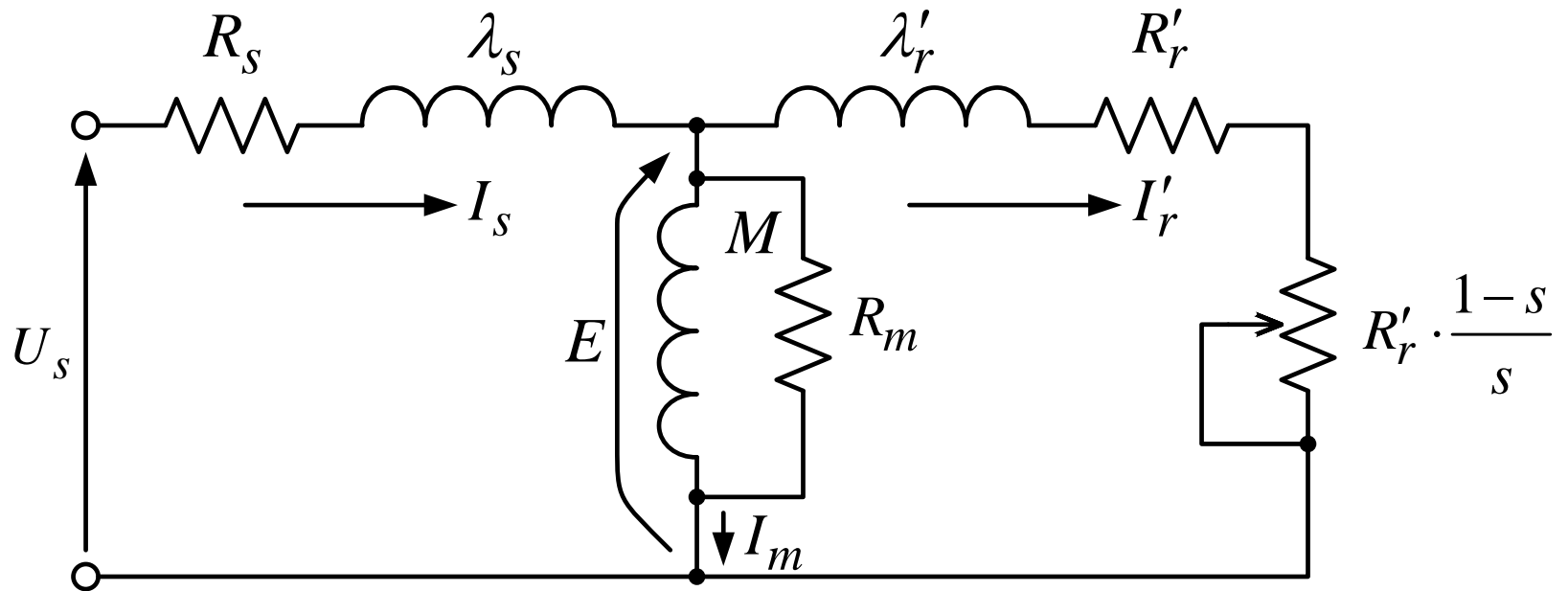
**ROTOR ASINHRONOG MOTORA NAJČEŠĆE IMA ZAKOŠENE ŽLEBOVE
DA BI SE MINIMIZIRALE PULSACIJE MOMENTA**

POPREČNI PRESEK ASINHRONOG MOTORA



STATIKA POGONA

Ekvivalentna šema motora (po fazi).



Rotorske veličine su svedene na stator.

Ostale karakteristične veličine:

f_s [Hz] - statorska učestanost;

f_r [Hz] - rotorska učestanost;

$\omega_s = 2\pi \cdot f_s$ [rad.el./s] - kružna učestanost statora ili električna sinhrona brzina ili brzina obrtnog polja (električna)

$\omega_r = 2\pi \cdot f_r$ [rad.el./s] - kružna učestanost rotora;
U knjizi: apsolutno klizanje

$\omega = \omega_s - \omega_r$ [rad.el./s] - ugaona brzina (električna);

P

- broj pari polova;

$\omega_m = \omega / P$ [rad.meh./s = rad/s] - mehanička ugaona brzina;

$s = \omega_r / \omega_s = (\omega_s - \omega) / \omega_s$ - klizanje.

BAZNE VREDNOSTI

$U_b = U_{sn}$ – nominalna efektivna vrednost faznog napona

$I_b = I_{sn}$ – nominalna efektivna vrednost fazne struje

$\omega_b = 2\pi f_{sn}$ – nominalna kružna učestanost (električna)

$$Z_b = U_b / I_b$$

$$P_b = q U_b I_b = 3 U_b I_b \quad q - \text{broj faza} = 3$$

$$M_b = P_b / (\omega_b / P)$$

TOKOVI SNAGE

$$P_s = 3 \cdot U_s \cdot I_s \cdot \cos \varphi_s \quad - \text{ snaga statora, snaga uzeta iz izvora;}$$

$$P_{sCu} = 3 \cdot R_s \cdot I_s^2 \quad - \text{ snaga gubitaka u bakru statora;}$$

$$P_{Fe} = 3 \cdot E^2 / R_m \quad - \text{ snaga gubitaka u gvožđu} \quad \left(\lim_{R_m \rightarrow \infty} P_{Fe} \rightarrow 0 \right)$$

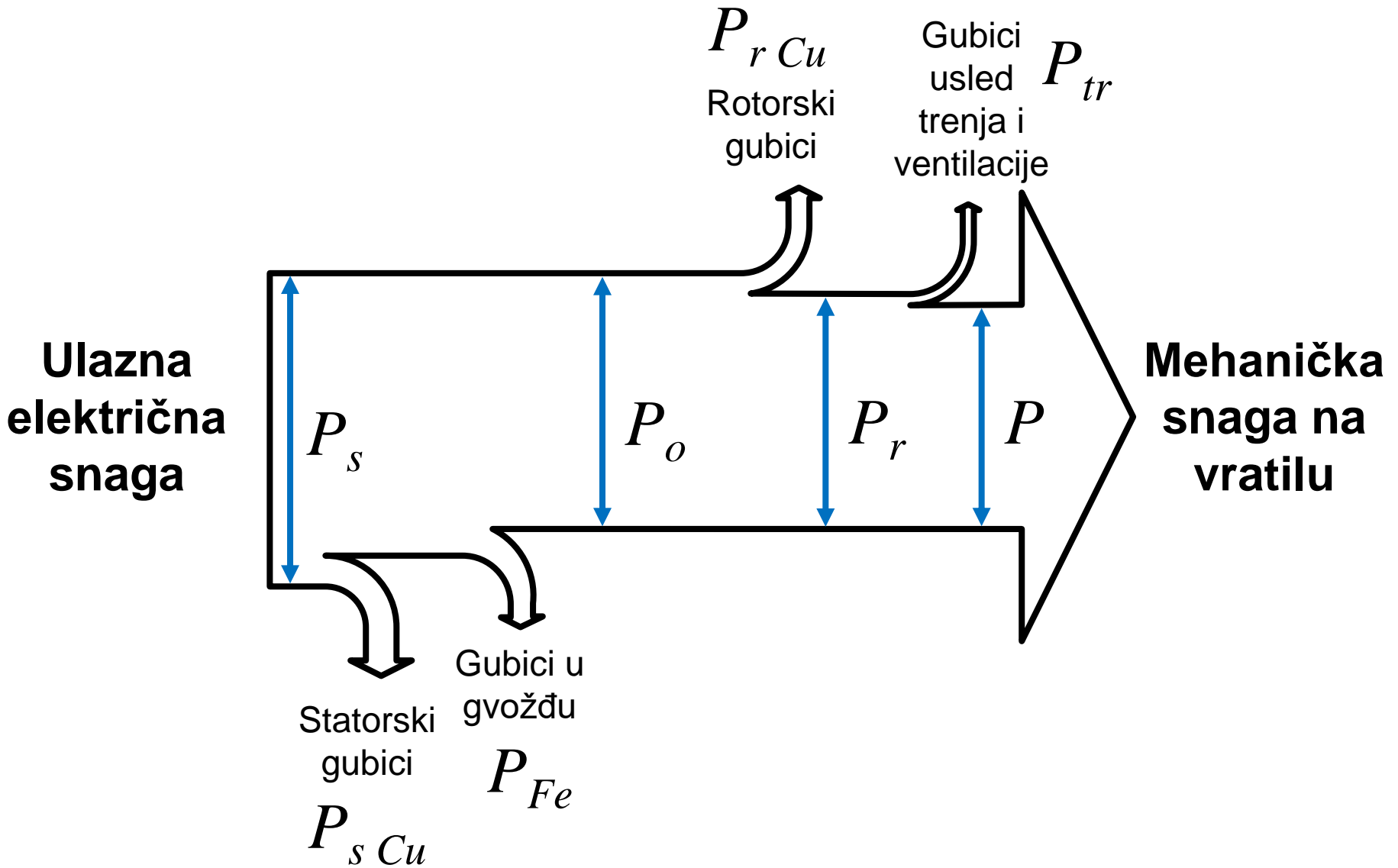
$$P_o = 3 \cdot (R'_r / s) \cdot (I'_r)^2 \quad - \text{ snaga obrtnog magnetnog polja;}$$

$$P_{rCu} = 3 \cdot R'_r \cdot (I'_r)^2 = s \cdot P_o \quad - \text{ snaga gubitaka u bakru rotora;}$$

$$P_r = P_o - P_{rCu} = 3 \cdot R'_r \cdot \frac{1-s}{s} \cdot (I'_r)^2 = (1-s) \cdot P_o \quad - \text{ mehanička snaga;}$$

$$P_{tr} \quad - \text{ snaga gubitaka na trenje i ventilaciju;}$$

$$P = P_r - P_{tr} \quad - \text{ korisna mehanička snaga.}$$



Napomena: Snaga P_{tr} prenosi se u opterećenje!

Elektromagnetni moment:

$$\begin{aligned} M_e &= \frac{P_r}{\omega_m} = 3 \cdot R'_r \cdot \underbrace{\frac{1-s}{s} \cdot (I'_r)^2}_{P_r} \cdot \frac{1}{\omega_m} = \\ &= 3 \cdot R'_r \frac{1-s}{s} \cdot (I'_r)^2 \cdot \left(\frac{\omega_s}{\omega} \right) \cdot \frac{P}{\omega} = \\ &= 3 \cdot P \cdot \frac{R'_r}{s} \cdot \frac{(I'_r)^2}{\omega_s} = 3 \cdot P \cdot R'_r \frac{(I'_r)^2}{\omega_r} \end{aligned}$$

MEHANIČKA KARAKTERISTIKA (statička karakteristika momenta)

U PREDSTOJEĆOJ ANALIZI PRETPOSTAVIMO DA JE $E = \text{const.}$

$$M_e = M_e(\omega)$$

$$|I'_r(s)| = \frac{|E|}{\sqrt{(R'_r/s)^2 + (\omega_s \cdot \lambda'_r)^2}}$$

$$M_e(s) = 3 \cdot P \cdot \frac{E^2}{\omega_s} \frac{R'_r/s}{(R'_r/s)^2 + (\omega_s \cdot \lambda'_r)^2}$$

$$M_e(\omega_r) = 3 \cdot P \cdot \left(\frac{E}{\omega_s} \right)^2 \frac{\omega_r \cdot R'_r}{(R'_r)^2 + (\omega_r \cdot \lambda'_r)^2}$$

Funkcija $M_e(s)$ ima ekstremum koji se može naći iz:

$$\frac{dM_e(s)}{ds} = 0$$

Momenat u tački ekstremuma naziva se **PREVALNI MOMENAT** (M_p),
a odgovarajuće klizanje **PREVALNO KLIZANJE** (s_p).

$$s_p = \pm \frac{R'_r}{\omega_s \cdot \lambda'_r}; \quad M_p = \pm \frac{3 \cdot P}{2} \cdot \left(\frac{E}{\omega_s} \right)^2 \frac{1}{\lambda'_r}$$

KLOSS - ova FORMULA

$$\frac{M_e}{M_p} = \frac{2}{\frac{s}{s_p} + \frac{s_p}{s}} = \frac{2}{\frac{\omega_r}{\omega_{rp}} + \frac{\omega_{rp}}{\omega_r}}$$

Važno:

$$\omega_{rp} = \omega_s \cdot s_p = \pm \frac{R'_r}{\lambda'_r} = \text{const.}$$

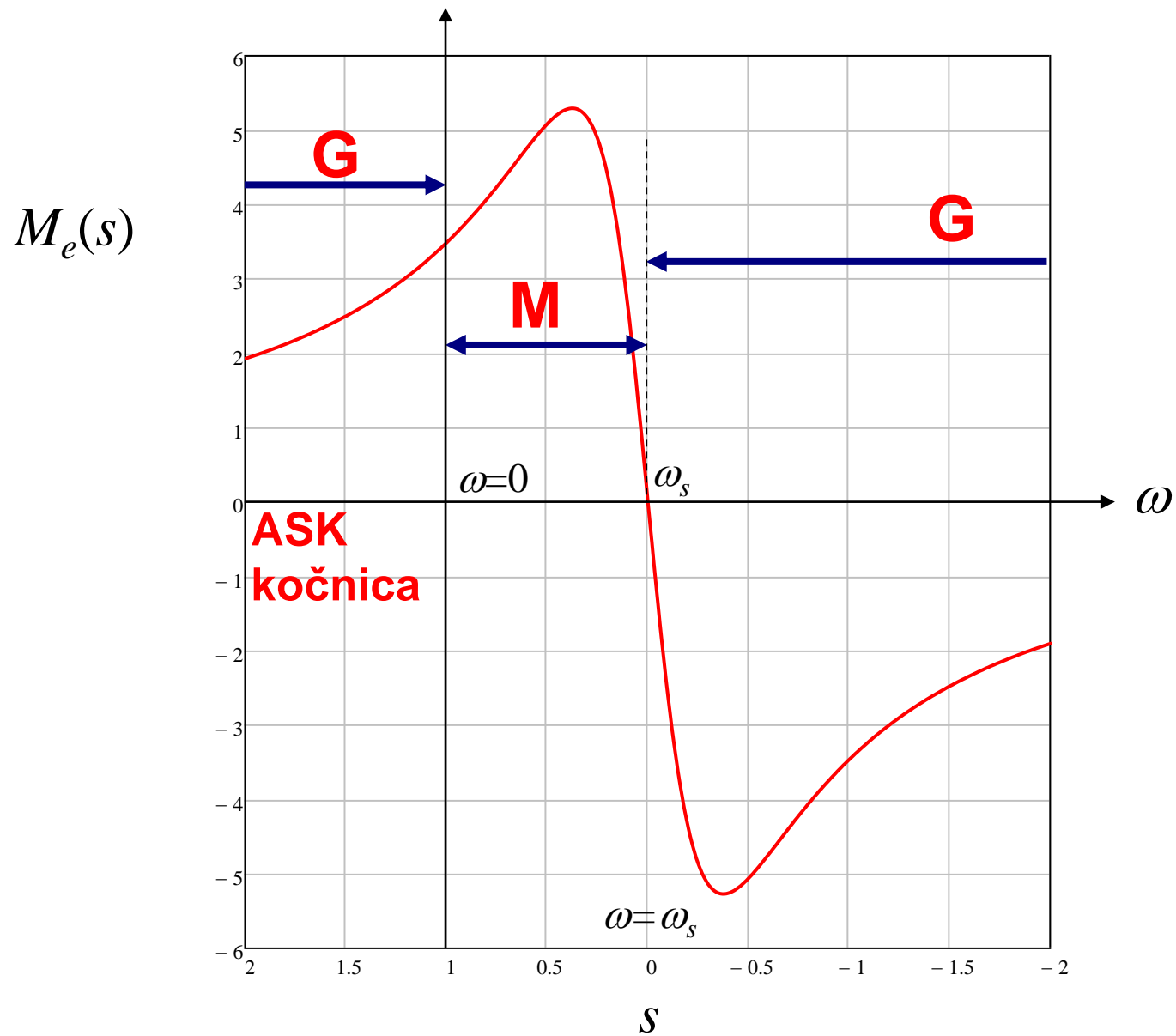
STATIČKE KARAKTERISTIKE STRUJA

$$\vec{I}_s = \vec{I}'_r + \vec{I}_m$$

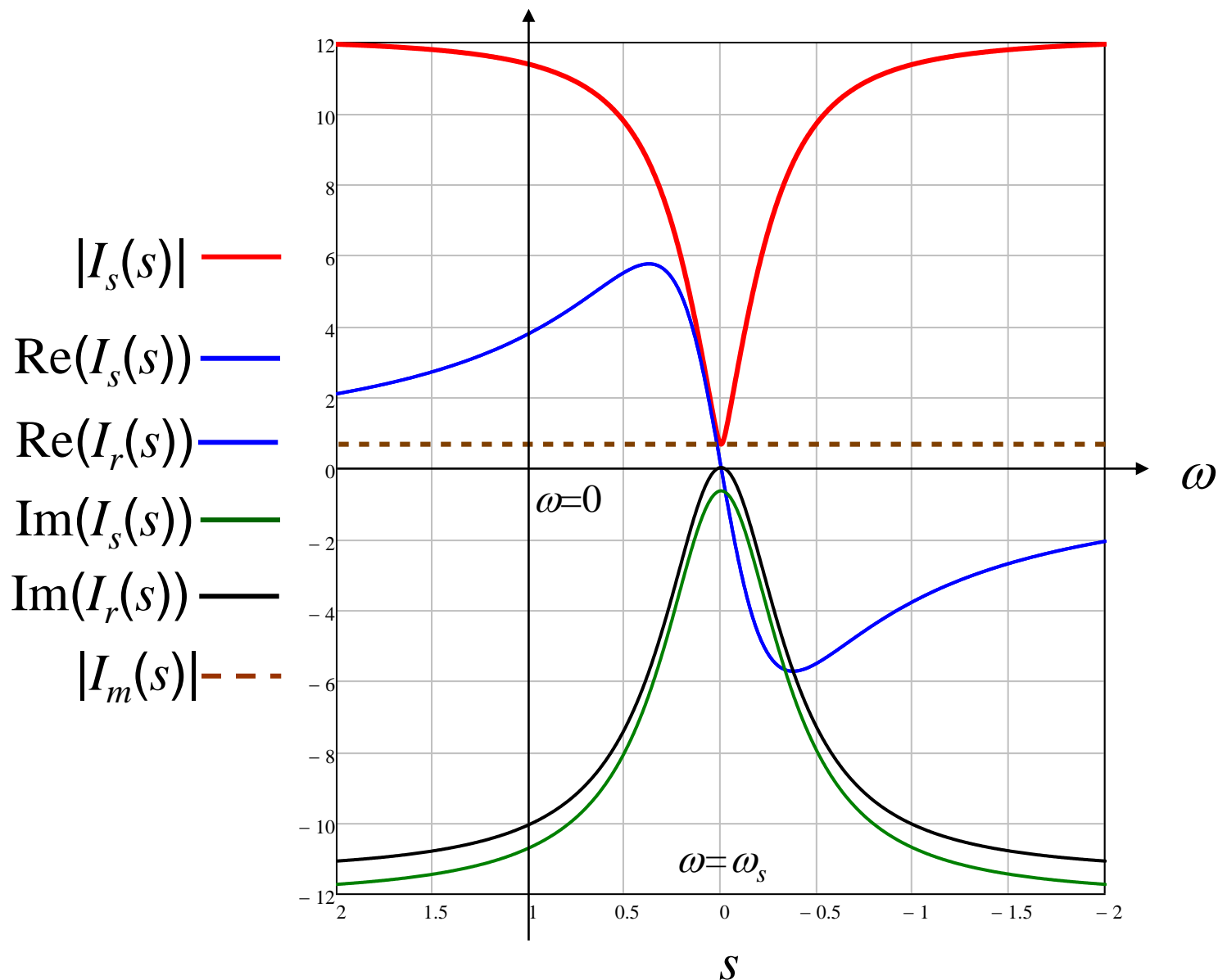
$$\vec{I}'_r = \frac{E \cdot R'_r / s}{(R'_r / s)^2 + (\omega_s \cdot \lambda'_r)^2} - j \frac{E \cdot \omega_s \cdot \lambda'_r}{(R'_r / s)^2 + (\omega_s \cdot \lambda'_r)^2} = I'_{ra} - jI'_{rr}$$

$$\vec{I}_m = -j \frac{E}{\omega_s \cdot M} \quad \text{za } P_{Fe} \approx 0 \quad \text{ili} \quad (R_m \rightarrow \infty)$$

Statička karakteristika momenta, pri $E=\text{const.}$



Statičke karakteristike struja, $E=\text{const.}$

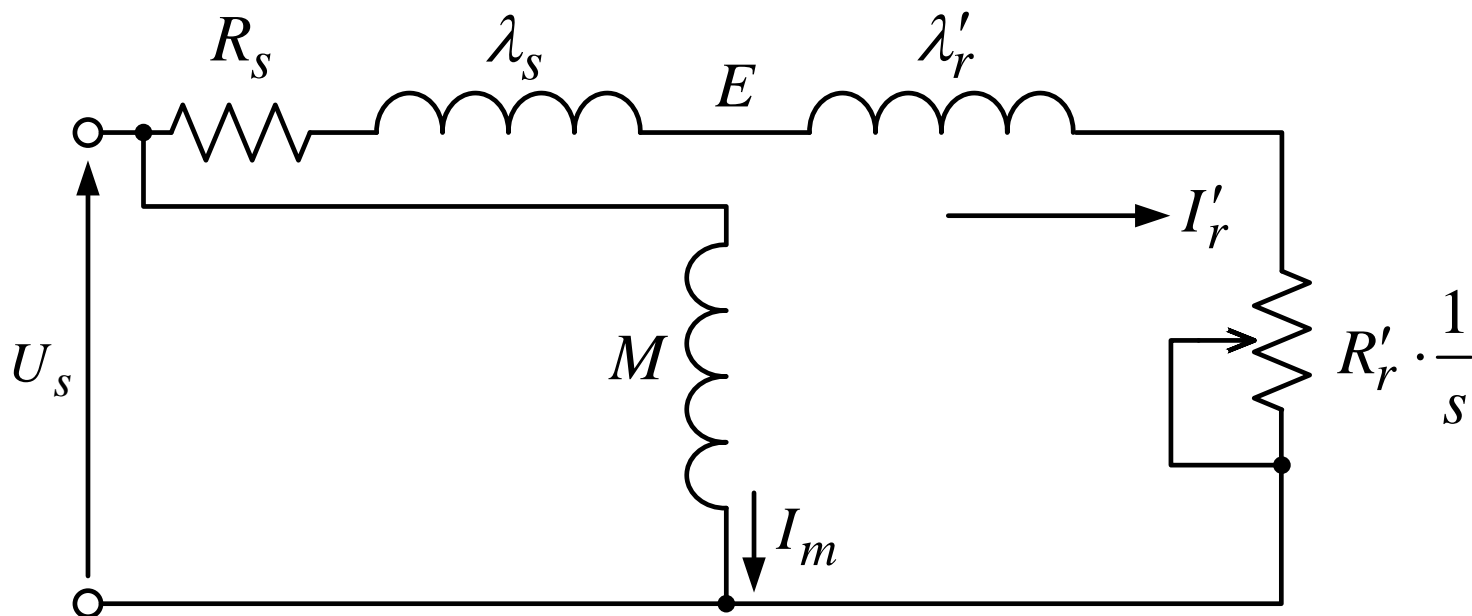


RAZMOTRIMO REALAN (realniji) SLUČAJ

$$E \neq \text{const.} \quad U_s = \text{const.}$$

Dve pretpostavke:

1. $P_{Fe} = 0 \Rightarrow R_m \rightarrow \infty$
2. $\omega_s^2 \cdot M^2 \gg R_s^2 + \omega_s^2 \cdot \lambda_s^2$ (sasvim realna pretpostavka)



Sa navedenim pretpostavkama se može napisati:

$$|\vec{I}'_r| = \frac{U_s}{\sqrt{(R_s + R'_r/s)^2 + \omega_s^2 \cdot (\lambda_s + \lambda'_r)^2}}$$

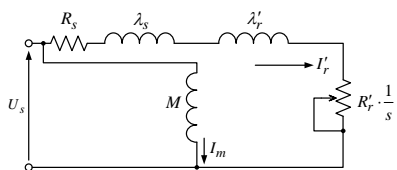
$$M_e = \frac{3 \cdot P \cdot U_s^2}{\omega_s} \cdot \frac{R'_r/s}{(R_s + R'_r/s)^2 + \omega_s^2 \cdot (\lambda_s + \lambda'_r)^2}$$

$$s_p = \pm \frac{R'_r}{\sqrt{R_s^2 + \omega_s^2 \cdot (\lambda_s + \lambda'_r)^2}} = f(\omega_s)$$

$$M_p = \pm \frac{3 \cdot P \cdot U_s^2}{2 \cdot \omega_s} \cdot \frac{1}{\sqrt{R_s^2 + \omega_s^2 \cdot (\lambda_s + \lambda'_r)^2} \pm R_s}$$

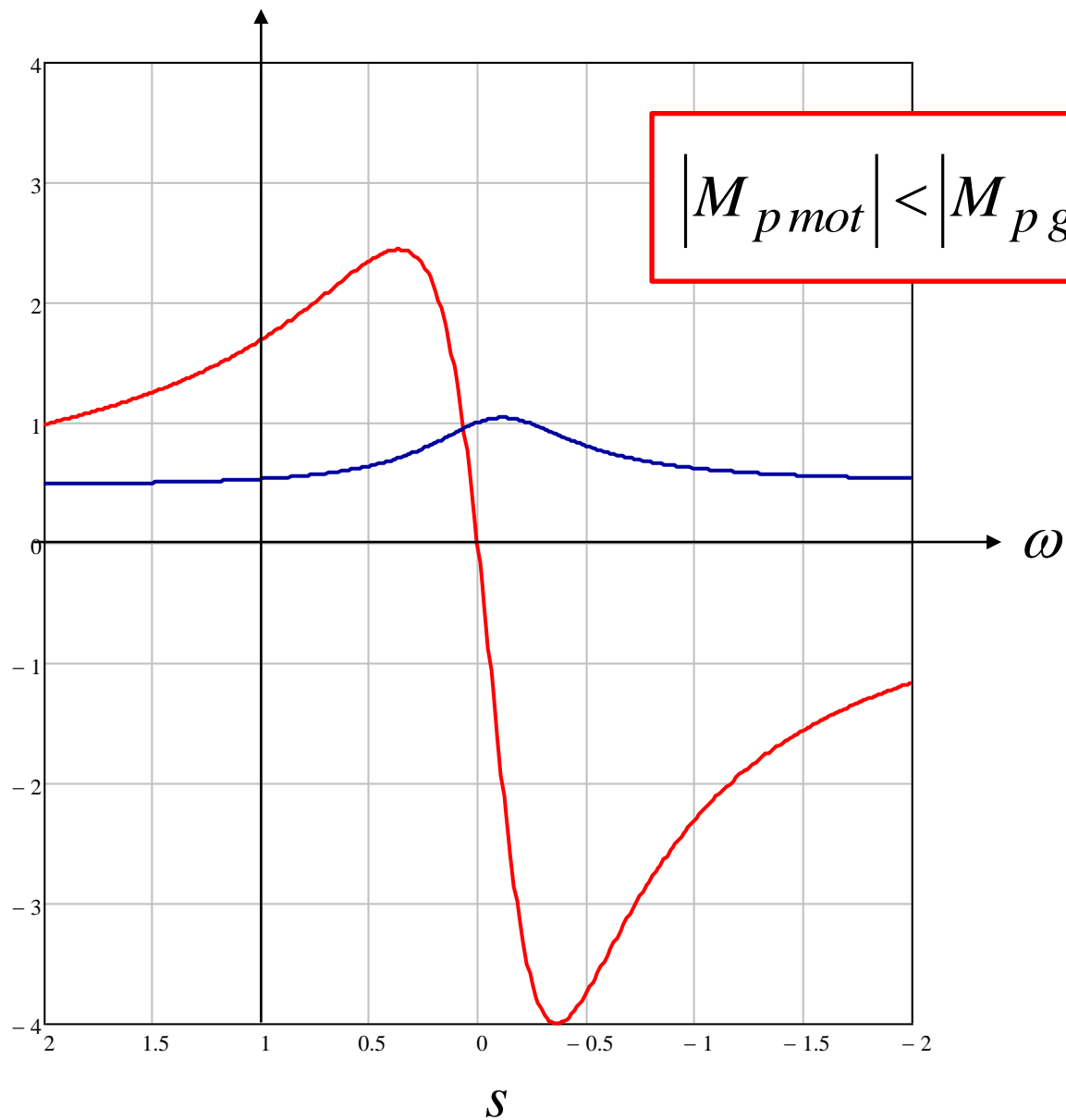
znaci: + za motorni režim;
- za generatorski režim.

Statička karakteristika momenta, pri $E \neq \text{const.}$, $U_s = U_{sn} = \text{const.}$

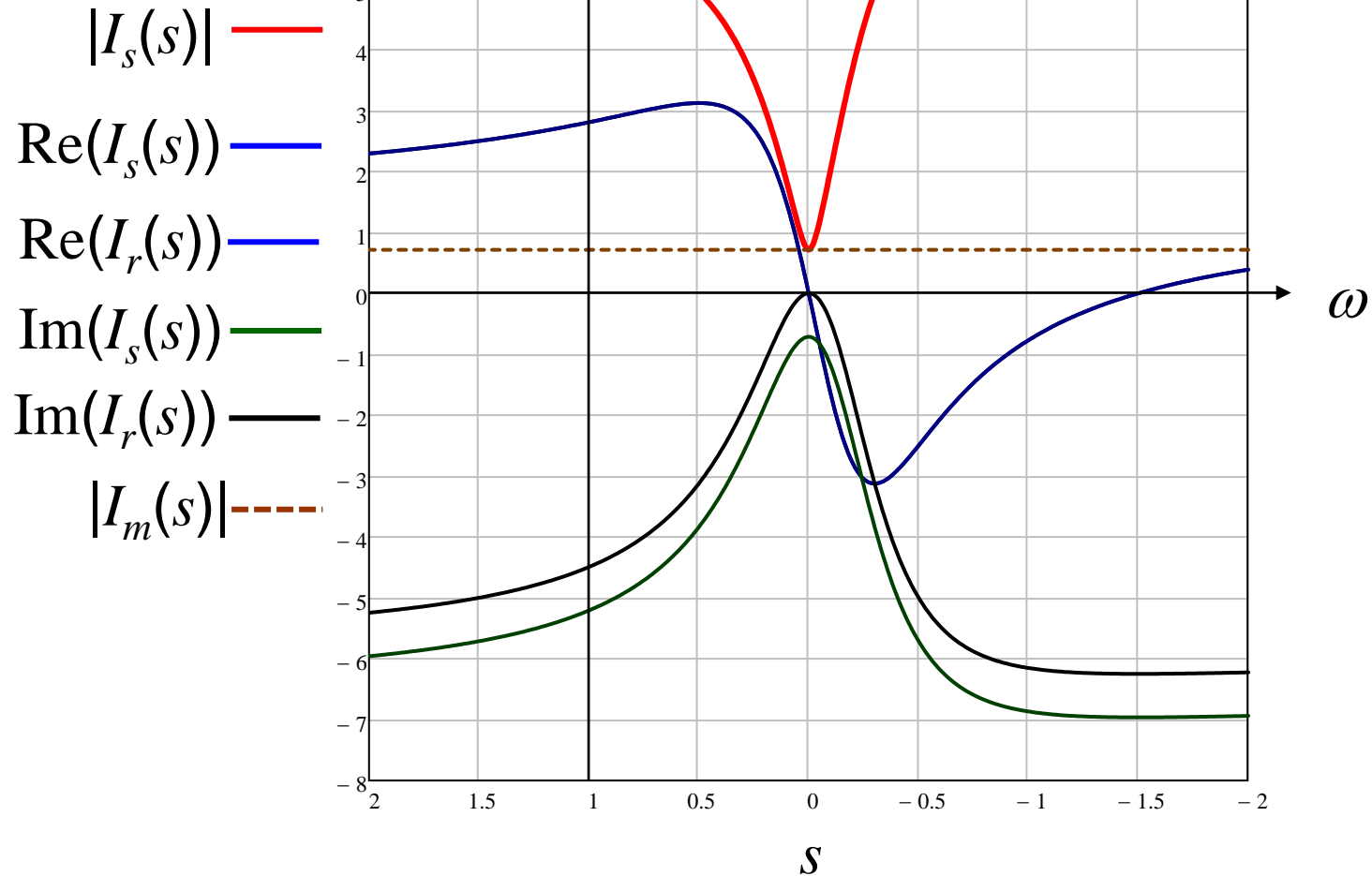
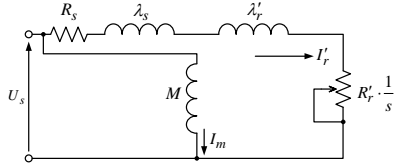


$M_e(s)$ ———

$|E(s)|$ ———



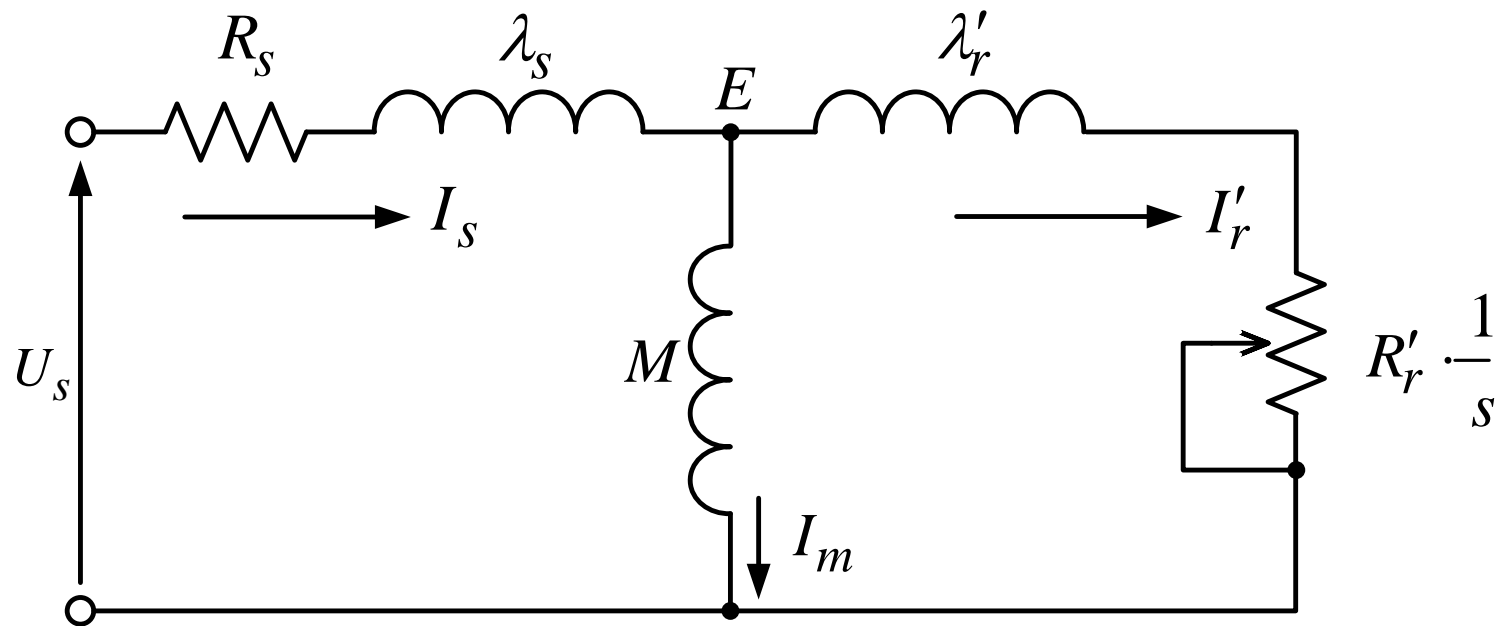
Statičke karakteristike struja, pri $E \neq \text{const.}$, $U_s = U_{sn} = \text{const.}$



RAZMOTRIMO SLUČAJ BEZ ZANEMARENJA INDUKTIVNOSTI MAGNEĆENJA

$$E \neq \text{const.} \quad U_s = \text{const.}$$

Jedina pretpostavka: $P_{Fe} = 0 \Rightarrow R_m \rightarrow \infty$



$$Z_s = R_s + j \cdot \omega_s \cdot \lambda_s \quad Z'_r(s) = R'_r / s + j \cdot \omega_s \cdot \lambda'_r$$

$$Z_m = j \cdot \omega_s \cdot M$$

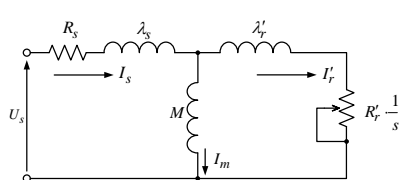
$$Z_e(s) = Z_s + Z_m \parallel Z'_r(s)$$

$$\vec{U}_s = U_s + j \cdot 0 = U_s \quad \vec{I}_s(s) = \frac{U_s}{Z_e(s)} \quad \vec{E}(s) = U_s - Z_s \cdot \vec{I}_s$$

$$\vec{I}_m(s) = \frac{\vec{E}(s)}{Z_m} \quad \vec{I}'_r(s) = \frac{\vec{E}(s)}{Z'_r(s)} = \vec{I}_s(s) - \vec{I}_m(s)$$

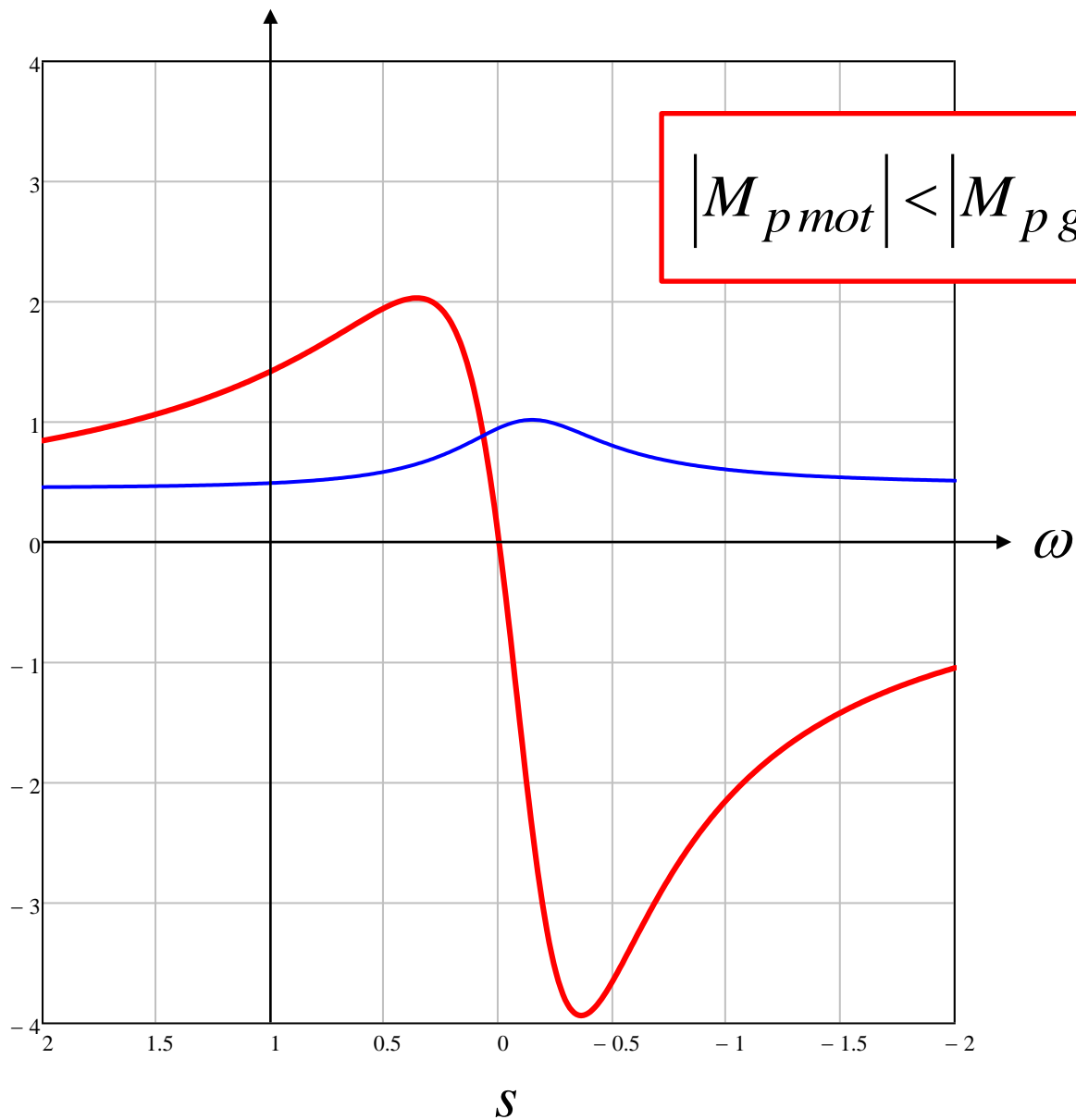
$$M_e = \frac{P_r}{\omega_m} = 3 \cdot P \cdot \frac{R'_r}{s} \cdot \frac{|\vec{I}'_r|^2}{\omega_s} = 3 \cdot P \cdot R'_r \frac{|\vec{I}'_r|^2}{\omega_r}$$

Statička karakteristika momenta, pri $E \neq \text{const.}$, $U_s = U_{sn} = \text{const.}$

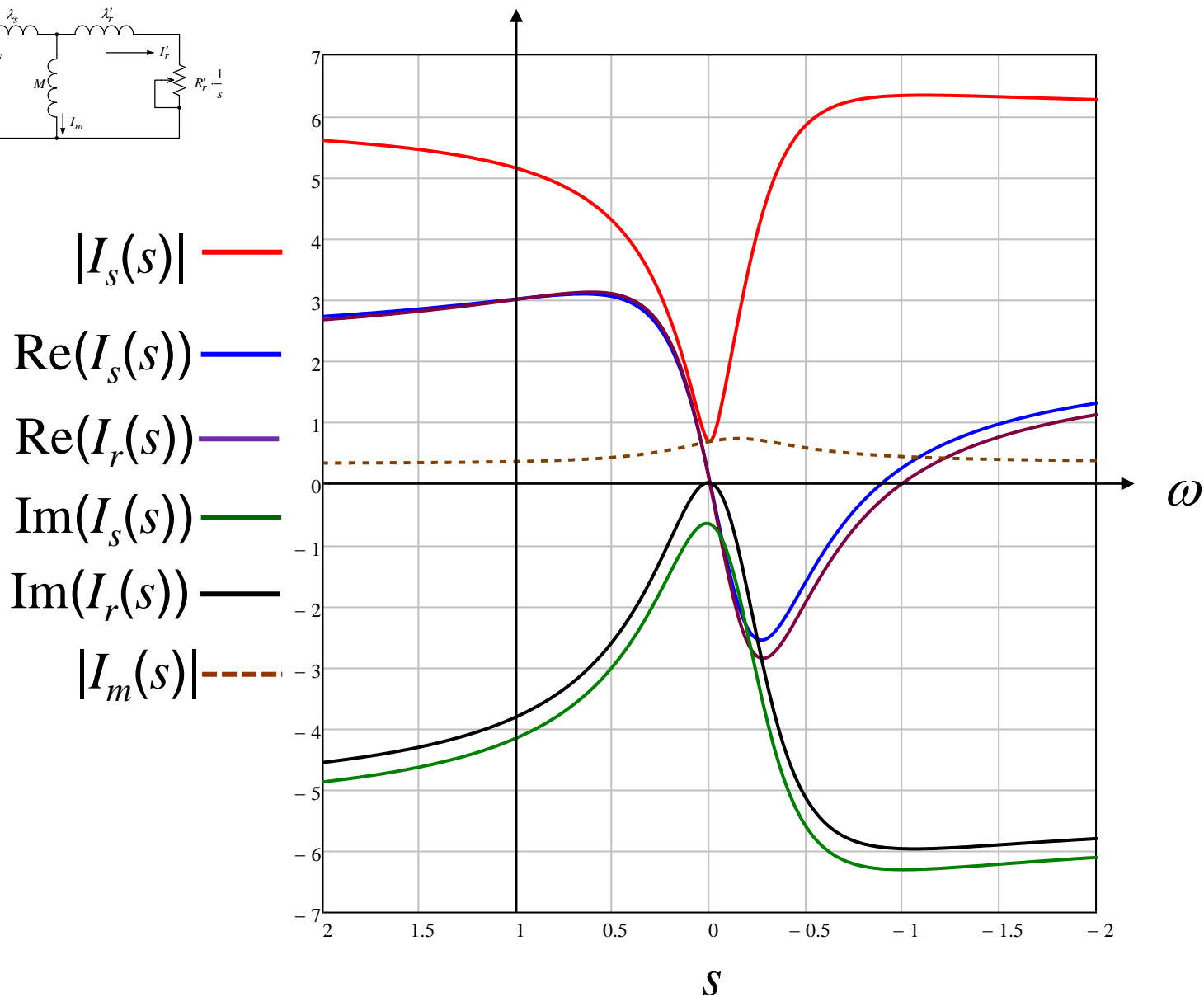
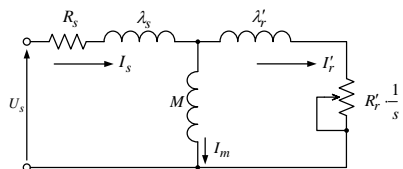


$M_e(s)$ — red line

$|E(s)|$ — blue line



Statičke karakteristike struja, pri $E \neq \text{const.}$, $U_s = U_{sn} = \text{const.}$



Kod velikih mašina može se smatrati: $R_s \approx 0 \quad M \gg (\lambda_s, \lambda'_r)$

Sada je:

$$M_e \approx \frac{3 \cdot P \cdot U_s^2}{\omega_s} \cdot \frac{R'_r / s}{(R'_r / s)^2 + \omega_s^2 \cdot (\lambda_s + \lambda'_r)^2}$$

Analički izraz
momenta
za Γ šemu

$$s_p = \pm \frac{R'_r}{\omega_s \cdot (\lambda_s + \lambda'_r)} \quad \omega_s \cdot s_p = \omega_{rp} = \pm \frac{R'_r}{(\lambda_s + \lambda'_r)}$$

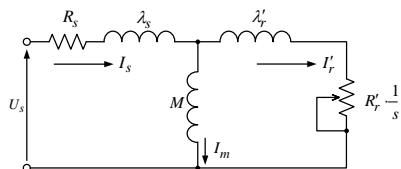
$$M_p = \pm \frac{3 \cdot P}{2} \cdot \left(\frac{U_s}{\omega_s} \right)^2 \cdot \frac{1}{\lambda_s + \lambda'_r}$$

Veoma slično kao kod $E=\text{const.}$ Može se izvesti **KLOSS** - ova formula.

$$\frac{M_e}{M_p} = \frac{2}{\frac{s}{s_p} + \frac{s_p}{s}} = \frac{2}{\frac{\omega_r}{\omega_{rp}} + \frac{\omega_{rp}}{\omega_r}}$$

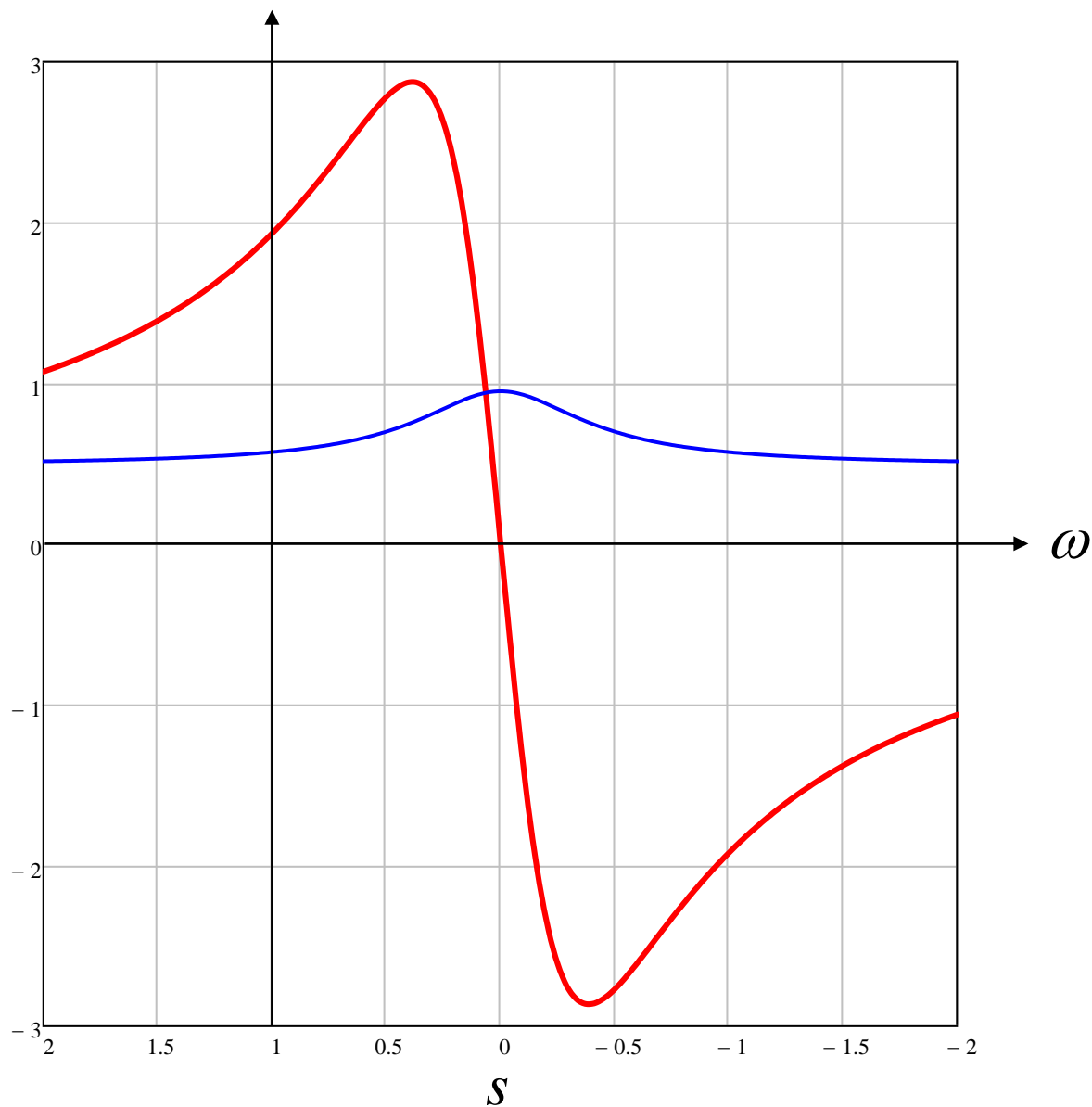
Statička karakteristika momenta, pri $E \neq \text{const.}$, $U_s = U_{sn} = \text{const.}$

$$R_s = 0$$



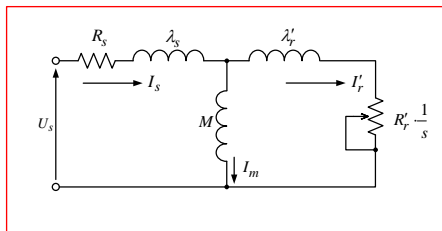
$$M_e(s) \quad \text{— red line —}$$

$$|E(s)| \quad \text{— blue line —}$$



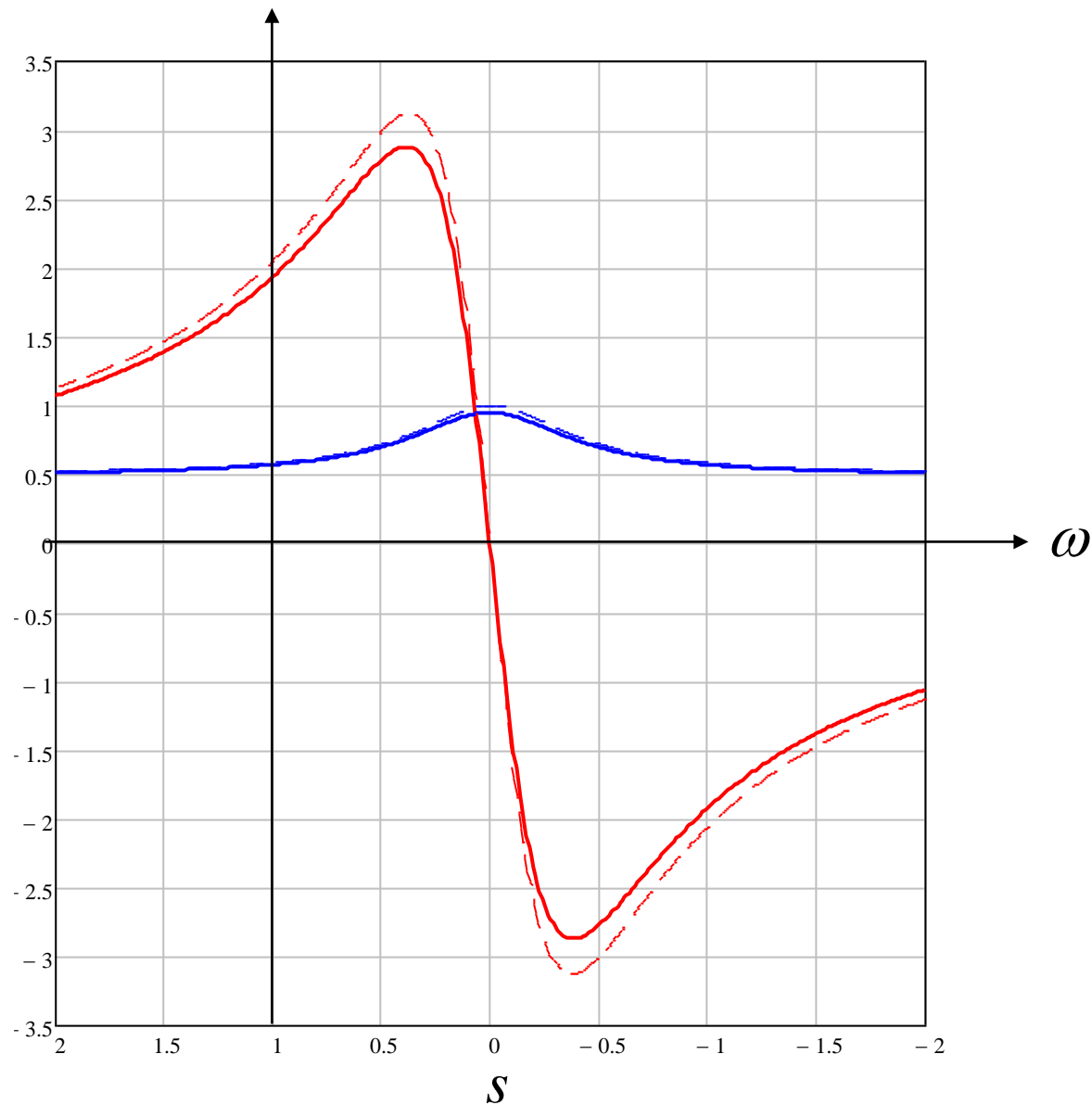
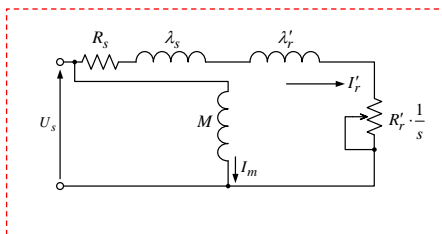
Statička karakteristika momenta, pri $E \neq \text{const.}$, $U_s = U_{sn} = \text{const.}$

$$R_s = 0$$



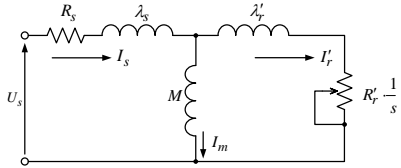
$$M_e(s) \quad \text{— red line —}$$

$$|E(s)| \quad \text{— blue line —}$$

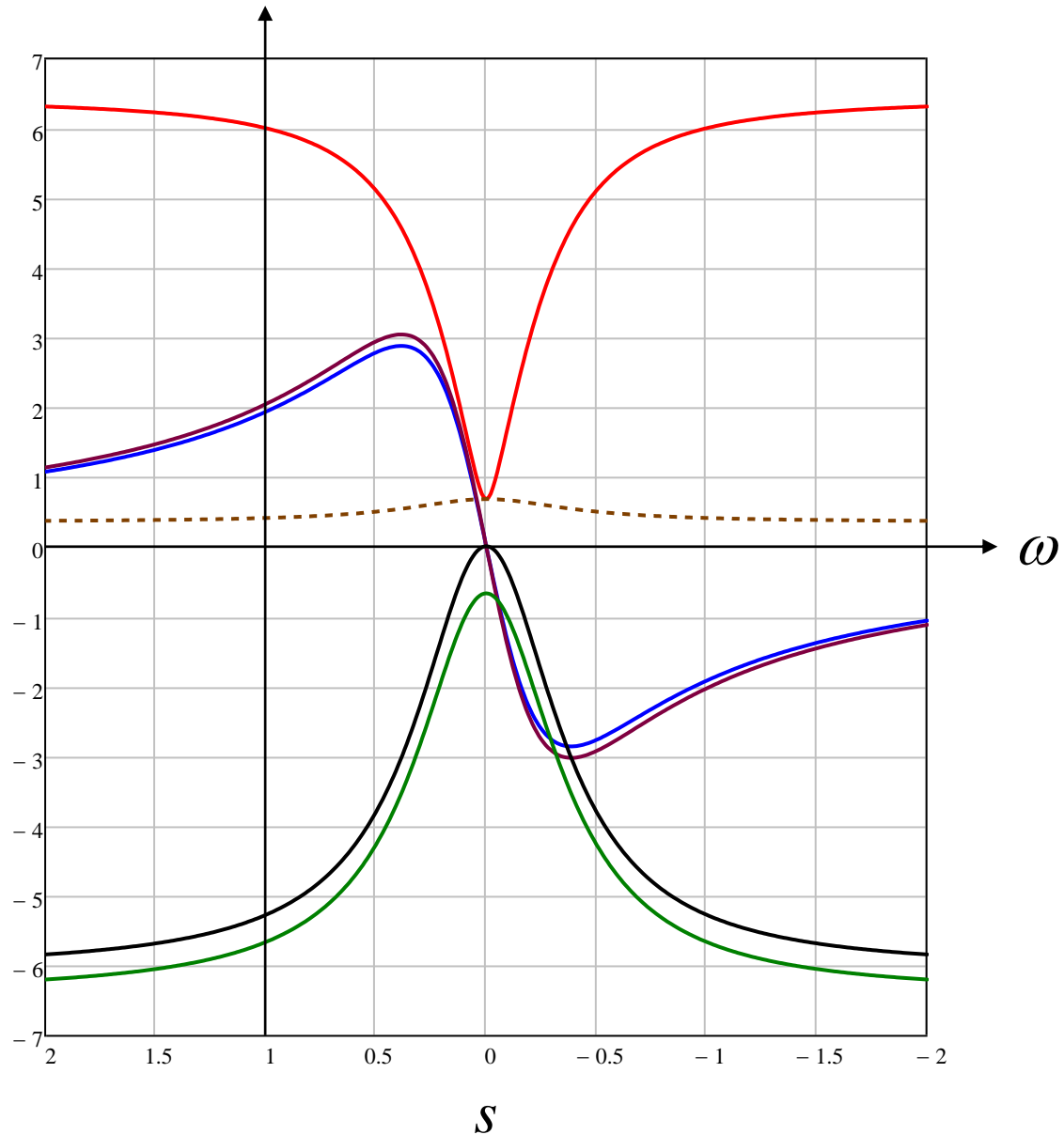


Statičke karakteristike struja, pri $E \neq \text{const.}$, $U_s = U_{sn} = \text{const.}$

$$R_s = 0$$



- $|I_s(s)|$ — red solid line
- $\text{Re}(I_s(s))$ — blue solid line
- $\text{Re}(I_r(s))$ — purple solid line
- $\text{Im}(I_s(s))$ — green solid line
- $\text{Im}(I_r(s))$ — black solid line
- $|I_m(s)|$ — brown dashed line

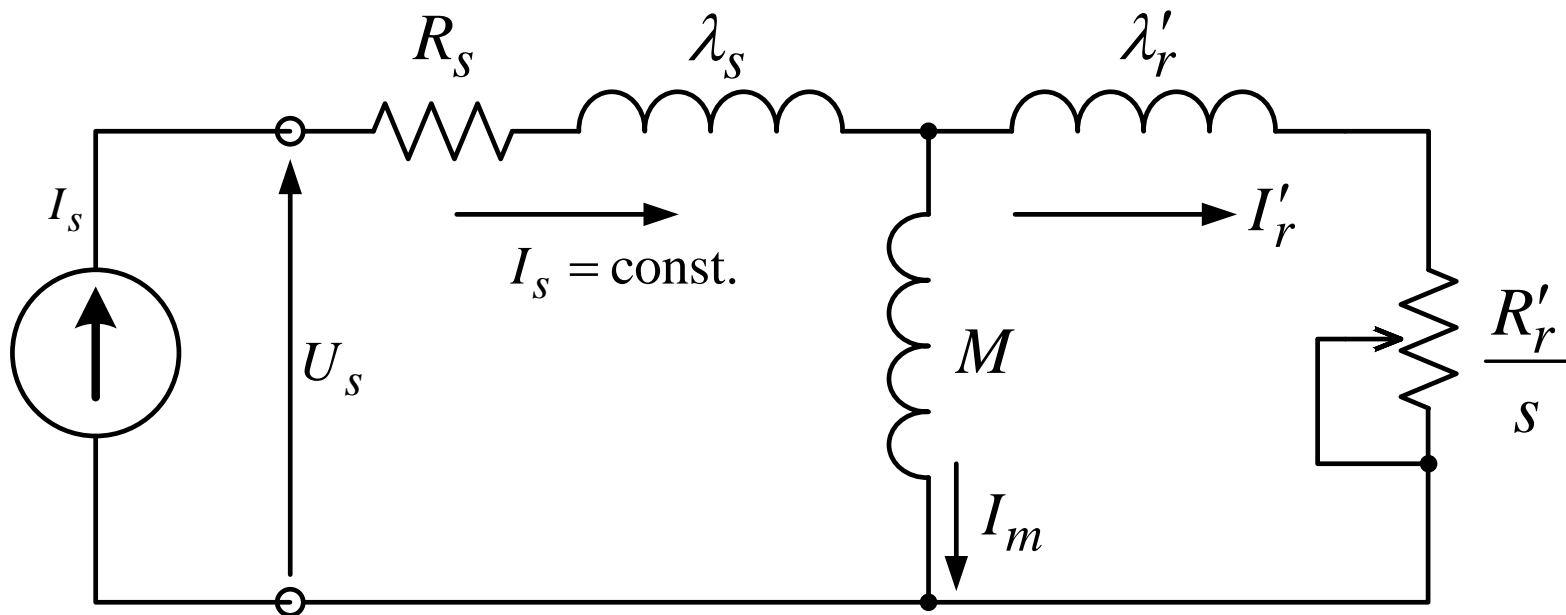


STRUJNO NAPAJANJE ASINHRONOG MOTORA

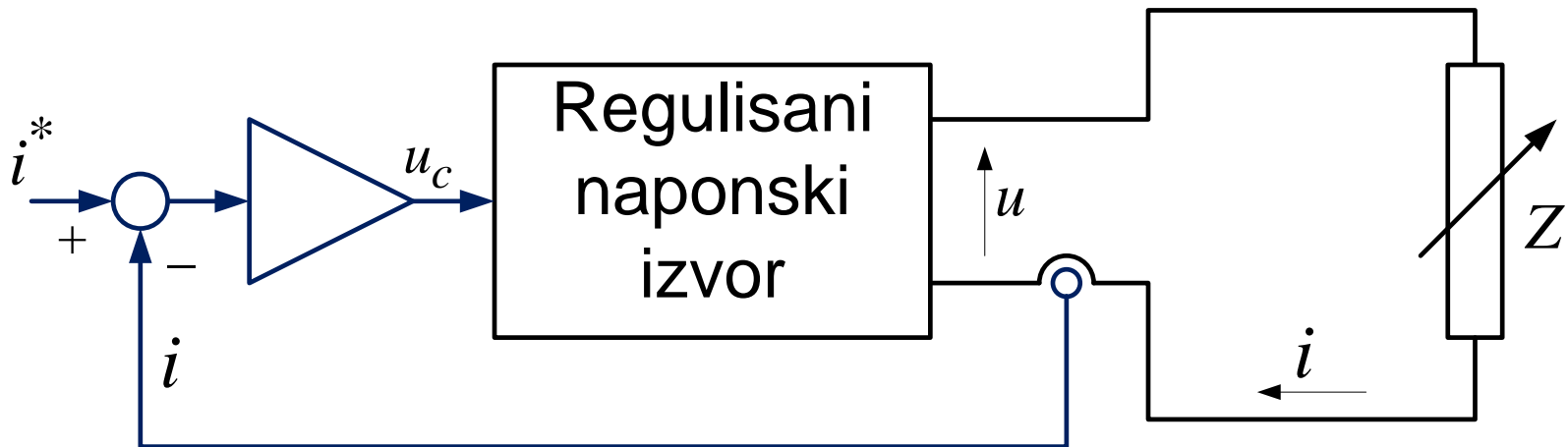
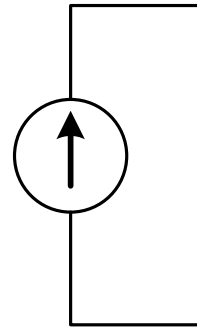
$$I_s = \text{const.}$$

Ovakav način napajanja pruža nove mogućnosti kod napajanja iz invertora.

U analizi zanemarujemo gubitke u gvožđu. $P_{Fe} = 0$



Praktična realizacija strujnog generatora



Statička karakteristika
momenta se izvodi iz:

$$\vec{I}_s = \vec{I}_m + \vec{I}'_r$$

$$\vec{I}_m \cdot Z_m = \vec{I}'_r \cdot Z_r$$

$$\vec{I}'_r = \frac{Z_m}{Z'_r + Z_m} \cdot \vec{I}_s = \frac{j \cdot \omega_s \cdot M}{R'_r/s + j \cdot \omega_s \cdot (M + \lambda'_r)} \cdot \vec{I}_s$$

$$|\vec{I}'_r|^2 = \frac{(\omega_s \cdot M)^2}{(R'_r/s)^2 + \omega_s^2 \cdot (M + \lambda'_r)^2} \cdot |\vec{I}_s|^2 = \frac{(\omega_r \cdot M)^2}{(R'_r)^2 + \omega_r^2 \cdot (M + \lambda'_r)^2} \cdot |\vec{I}_s|^2$$

$$M_e = 3 \cdot P \cdot \frac{R'_r}{s} \frac{\omega_s \cdot M^2}{\left(\frac{R'_r}{s}\right)^2 + \omega_s^2 \cdot (M + \lambda'_r)^2} \cdot |\vec{I}_s|^2 =$$

$$= 3 \cdot P \cdot \frac{\omega_r \cdot R'_r \cdot M^2}{(R'_r)^2 + \omega_r^2 \cdot (M + \lambda'_r)^2} \cdot |\vec{I}_s|^2 = f(\omega_r, I_s)$$

Funkcija $M_e(s)$ ima ekstremum koji se može naći iz: $\frac{dM_e(s)}{ds} = 0$

Vrednosti prevalnog momenta i klizanja pri strujnom napajanju su:

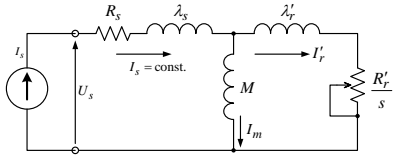
$$M_p = \pm \frac{3 \cdot P}{2} \cdot |\vec{I}_s|^2 \cdot \frac{M^2}{M + \lambda'_r} \quad s_p = \pm \frac{R'_r}{\omega_s \cdot (M + \lambda'_r)}$$

Pomoću ovih prevalnih vrednosti može se izvesti odgovarajuća KLOSS-ova formula.

Mehanička karakteristika kod strujnog napajanja ima isti oblik kao i kod naponskog napajanja, ali se karakteristične vrednosti razlikuju.

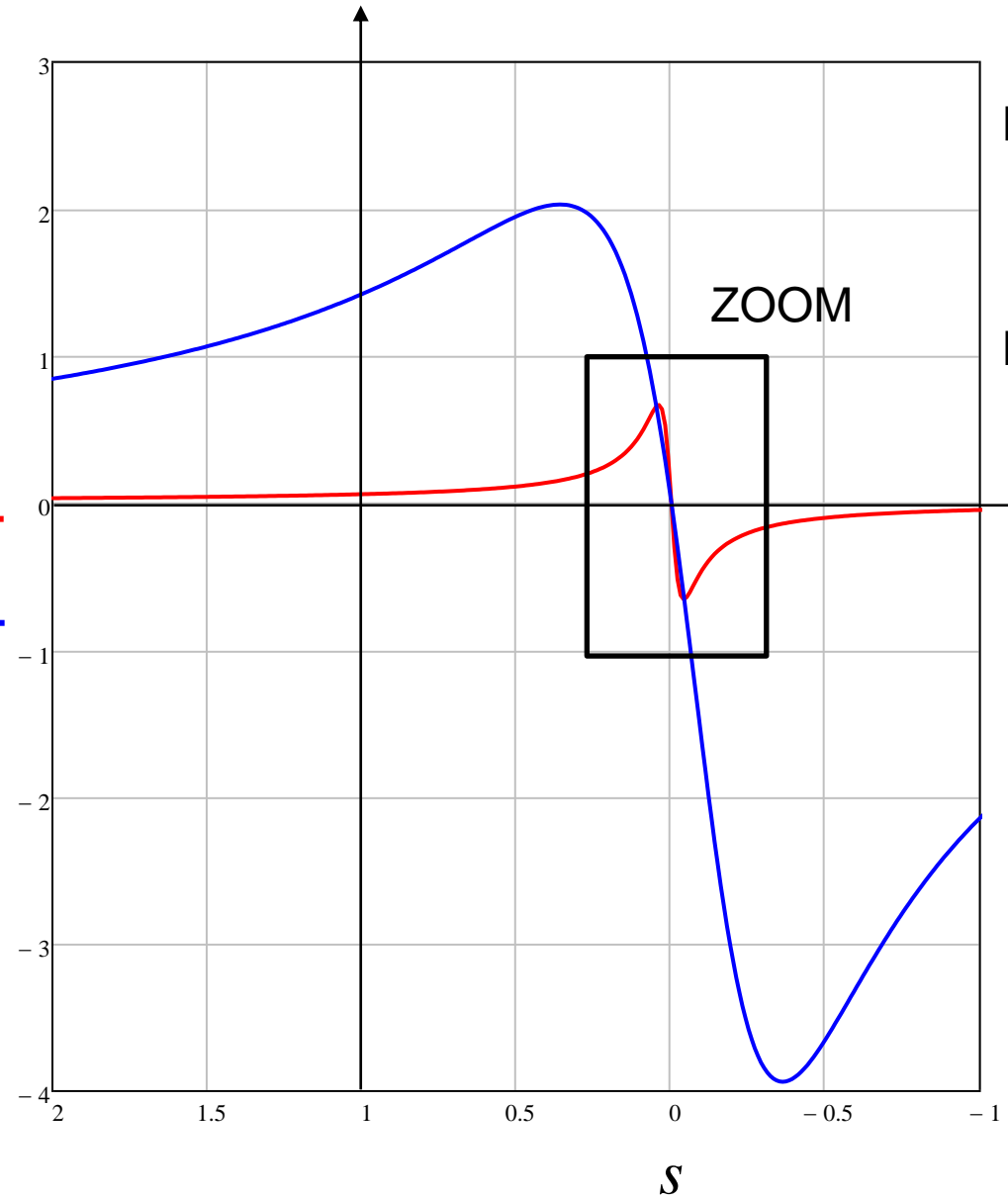
$$\left. \begin{array}{l} s_{p \text{ nap.}} > s_{p \text{ str.}} \\ M_{p \text{ nap.}} > M_{p \text{ str.}} \end{array} \right\} \text{ jer je } M \gg \lambda_s + \lambda'_r$$

Statička karakteristika momenta, pri strujnom napajanju $I_s = I_{sn}$



$M_{e\ str}(s)$ ———

$M_{e\ nap}(s)$ ———



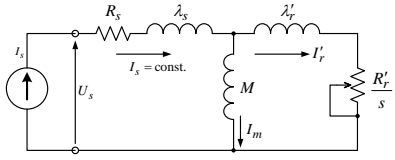
naponska karakteristika
 ——— $U_s = 1$

strujna karakteristika
 ——— $I_s = 1$

ω

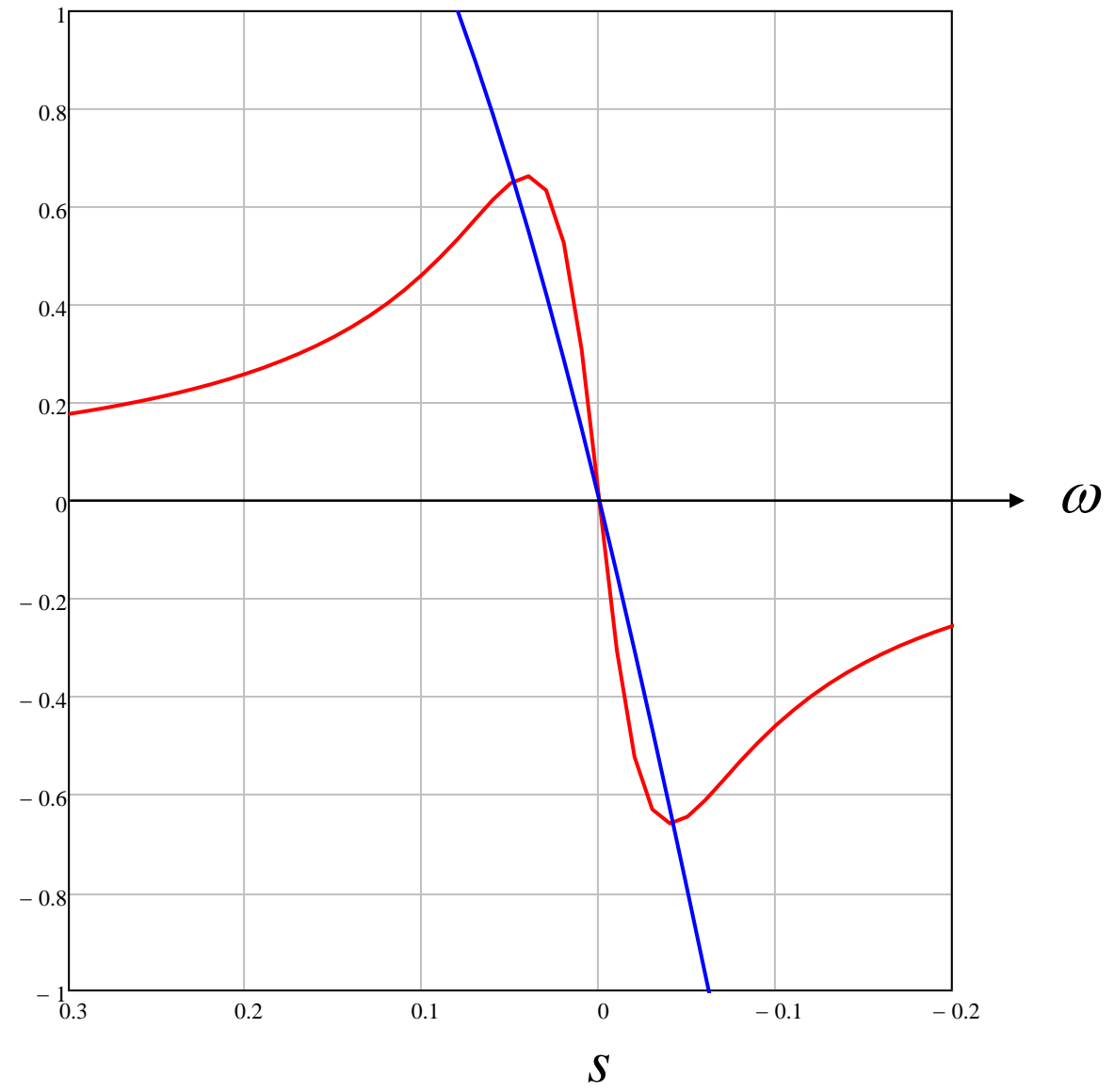
s

Statička karakteristika momenta, pri strujnom napajanju $I_s = I_{sn}$

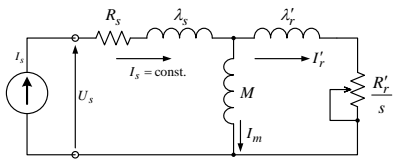


$M_{e\ str}(s)$ ——— red line

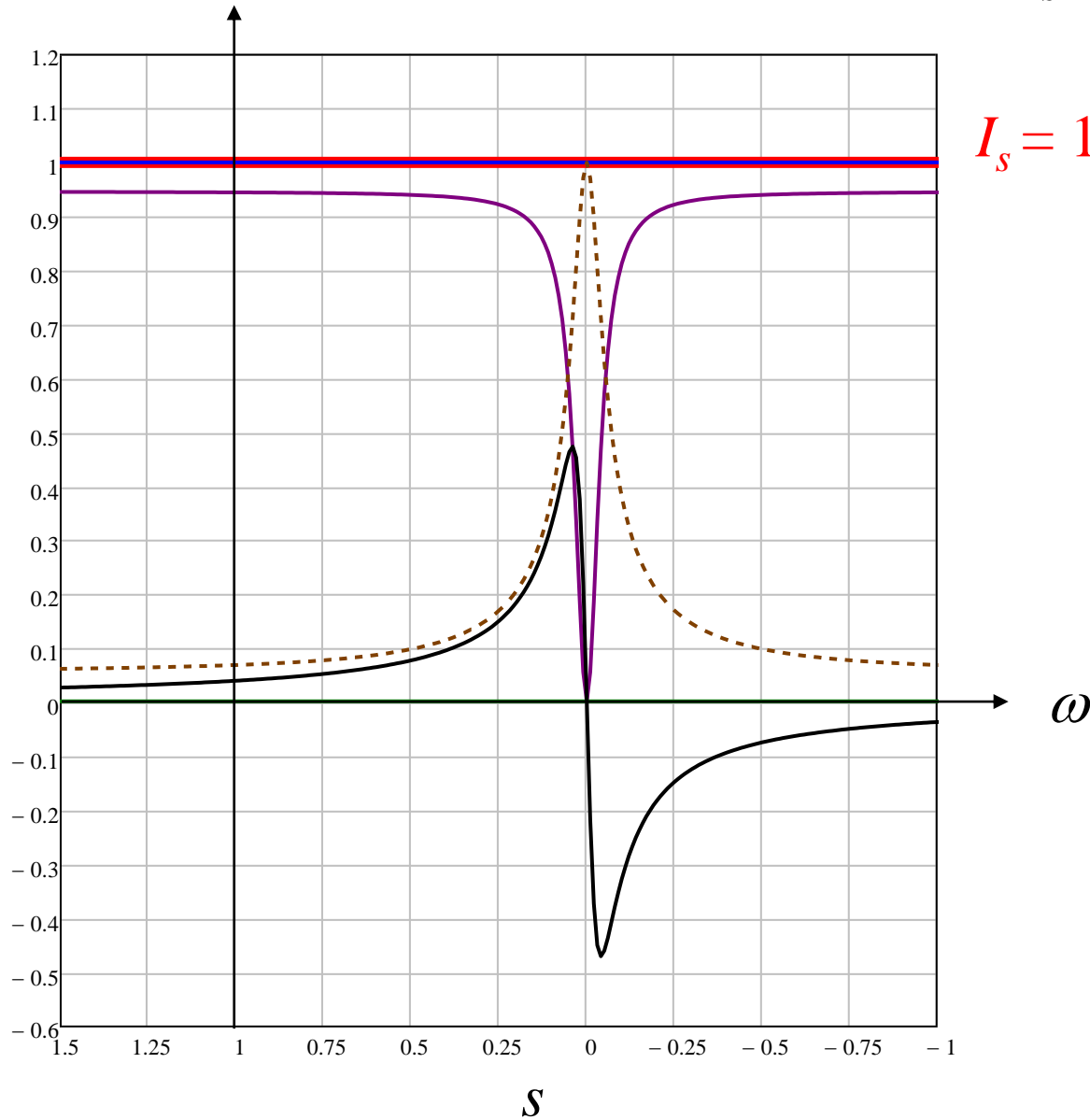
$M_{e\ nap}(s)$ ——— blue line



Statičke karakteristike struja, pri strujnom napajanju $I_s = I_{sn}$



- $|I_s(s)|$ — —
- $\text{Re}(I_s(s))$ — —
- $\text{Re}(I_r(s))$ — —
- $\text{Im}(I_s(s))$ — —
- $\text{Im}(I_r(s))$ — —
- $|I_m(s)|$ — - - -



STATIČKA KARAKTERISTIKA NAPONA

$$\vec{U}_s(s) = \vec{I}_s \cdot Z_{ekv}(s)$$

$$\begin{aligned} Z_{ekv}(s) &= Z_s + Z_m \parallel Z'_r(s) = \\ &= R_s + j \cdot \omega_s \cdot \lambda_s + \frac{j \cdot \omega_s \cdot M \cdot \left[(R'_r/s) + j \cdot \omega_s \cdot \lambda'_r \right]}{(R'_r/s) + j \cdot \omega_s \cdot (M + \lambda'_r)} \end{aligned}$$

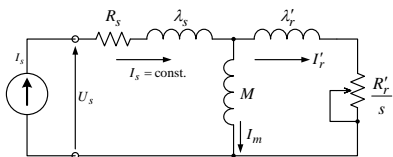
Uz uvažavanje $\lambda'_r \ll M \Rightarrow M - \lambda'_r \approx M$ može se napisati:

$$Z_{ekv}(s) = R_s + \frac{s \cdot \omega_s^2 \cdot M^2 \cdot R'_r}{(R'_r)^2 + s^2 \cdot (\omega_s \cdot M)^2} + j \cdot \omega_s \cdot \left(\lambda_s + \frac{s^2 \cdot \omega_s^2 \cdot \lambda'_r \cdot M^2 + M \cdot (R'_r)^2}{(R'_r)^2 + s^2 (\omega_s \cdot M)^2} \right)$$

$$\lim_{s \rightarrow 0} Z_{ekv} = R_s + j \cdot \omega_s \cdot (\lambda_s + M)$$

$$\lim_{s \rightarrow 1} Z_{ekv} \approx R_s + R'_r + j \cdot \omega_s \cdot (\lambda_s + \lambda'_r) \quad \text{jer je:} \quad R'_r \ll \omega_s \cdot M$$
$$(R'_r)^2 \ll (\omega_s \cdot M)^2$$

Statička karakteristika napona statora kod strujno napajanog asinhronog motora pri nominalnoj struji



$$I_s = I_{sn}$$

$$|U_s(s)|$$

